Improving Performance of a Path-Based Equivalence Checker using Counter-Examples

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***Abstract*—Path-based equivalence checkers (PBECs) have been successfully applied for verification of programs from diverse domains and at various stages of high-level synthesis. These verifiers can be sound but not complete. Therefore, non-equivalence cases require further investigation of the two programs being compared by some human expert. In this work, we show how a counter-trace (*cTrace*) can be generated in the case of non-equivalence reported by the PBEC. We show how a Bounded Model Checker (CBMC) can be used to find suitable initialization values for input variables (i.e., a counter-example) for a given *cTrace*. With our counter-example generation framework, we show how a strong non-equivalence decision can be taken in a PBEC. We also show that some false negative cases of the PBEC can also be revealed using this framework. Experimental results demonstrate the usefulness of our method.**

***Index Terms*—Equivalence Checking, Finite State Machine with Datapath (FSMD), CBMC, Counter-example Generation.**

I. Introduction

High-level synthesis (HLS) is the process of translating a behavioral description into a Register Transfer Level (RTL) description [1]. HLS tools are large and complex software systems and are very often written without formally proving their correctness. Many path-based approaches [2]–[8] have been proposed for verification of the scheduling phase of HLS where each behavior is represented by a finite state machine with datapaths (FSMD) [1]. In general, path-based approaches decompose each FSMD into a finite set of finite paths and the equivalence of FSMDs is established by showing path level equivalence between two FSMDs. In the case of non-equivalence, these approaches do not provide information sufficient for debugging the issue. A counter-example which will demonstrate the non-equivalence between the input behavior to HLS (i.e., source behavior) and the scheduled behavior generated by HLS (i.e., transformed behavior) will add significant value to the adoption of such PBECs. In this case, PBEC can report "Not equivalent" instead of "May Not be equivalent" Equivalence checking of programs is an undecidable problem in general. Therefore it is possible that a PBEC may produce a *false negative* result, i.e., a PBEC may report that two behaviors "May Not be equivalent" but these two behaviors are actually equivalent. The process of generating a counter-example helps to identify some false negative cases of a PBEC. Thus, a counter-example generation procedure helps to improve the performance of a PBEC.

Specifically, the contributions of the paper are as follows:

1) We show how the equivalence information of the enhanced value propagation (EVP) based PBEC [8] can be used to find a $cTrace$ in the case of non-equivalence reported by the PBEC.
2) We show how the CBMC [9] tool can be used to find a suitable counter-example for a given $cTrace$.
3) We show how to improve the performance of the PBEC using this counter-example generation framework.
4) An enhanced version of PBEC [8] after incorporating our counter-example generation scheme is also presented.

To the best of our knowledge, this is the first work which reports a $cTrace$ in the case of non-equivalence and uses it to produce a counter-example and improve the performance of PBECs during verification of the scheduling phase of HLS.

The rest of this paper is organized as follows. Section II describes an FSMD model and the EVP method. Section III focuses on *cTrace* generation. Section IV presents how that *cTrace* can be used to produce a counter-examples using CBMC. Section V and VI finally delve into how current PBECs can be enhanced by incorporating our counter-example generation technique. Experimental results are given in Sec. VII. Section VIII contains a summary of the related work. Section IX concludes the paper.

II. The FSMD model and path-based equivalence checking

This section briefly explains the FSMD model and the EVP method [8]. The details can be found in [5], [8].

An FSMD is defined as a 7-tuple $\langle Q, q_0, I, V, O, f : Q \times 2^S \rightarrow Q, h : Q \times 2^S \rightarrow U \rangle$, where Q is the finite set of states, q_0 is the reset state, I is the set of input variables, V is the set of storage variables, O is the set of output variables, f is the state transition function, h is the update function of the output and the storage variables. Here U represents a set of storage and output assignments and S represents a set of relations over arithmetic expressions and Boolean literals.

A computation of an FSMD is a finite walk from the reset state q_0 to itself, and q_0 should not occur in between. The papers [5], [8] breaks down an FSMD into smaller segments by introducing cutpoints so that each loop in an FSMD is cut at at least one cutpoint. A *path* α is a finite sequence of states from a cutpoint to another cutpoint without an intermediate occurrence of a cutpoint. The *condition of execution* R_α of a path α is a logical expression over $I \bigcup V$ such that R_α is satisfied by the (initial) data state of the path iff the path α is traversed. The *data transformation* r_α of a path α over V is the tuple $\langle s_\alpha, \theta_\alpha \rangle$; the first member s_α represents the value of the variables v_i after the execution of the path in terms of the initial data state of the path; the second member θ_α represents the output list along the path α. Two paths α and β are equivalent denoted by $\alpha \simeq \beta$ if $R_\alpha \equiv R_\beta$ and $r_\alpha = r_\beta$. A finite set of paths $P = \{\alpha_1, \alpha_2, \ldots, \alpha_k\}$ is said to be a path cover of an FSMD M if any computation μ of M can be looked upon as a concatenation of paths from P [10].

An FSMD M_0 is contained in another FSMD M_1, symbolically $M_0 \sqsubseteq M_1$, if there exists a finite path cover $P_0 = \{\alpha_1, \alpha_2, \ldots, \alpha_l\}$ of M_0 for which there exists a set $P_1 = \{\beta_1, \beta_2, \ldots, \beta_l\}$ of paths of M_1 such that $\alpha_i \simeq \beta_i$, $1 \leq i \leq l$. Two FSMDs M_0 and M_1 are said to be computationally equivalent ($M_0 \equiv M_1$), if $M_0 \sqsubseteq M_1$ and $M_1 \sqsubseteq M_0$.

The EVP method [8] is based on propagating the mismatched variable values (as *propagated vectors*) over a path to the subsequent paths until the values match or the final path segments end in the reset state without a match. During the course of equivalence checking of two behaviors, two paths, α and β say, (one from each behavior) are compared with respect to their corresponding propagated vectors for finding path equivalence. If $R_\beta \equiv R_\alpha$ and $r_\beta = r_\alpha$, then these paths are declared as unconditionally equivalent (U-equivalent, represented as $\alpha \simeq \beta$); if some mismatch is detected in data transformation, then they are declared to be conditionally equivalent (C-equivalent, represented as $\alpha \simeq_c \beta$), if their final state-pair always eventually lead to some U-equivalent paths; otherwise they are declared to be not equivalent.

III. Counter-trace generation

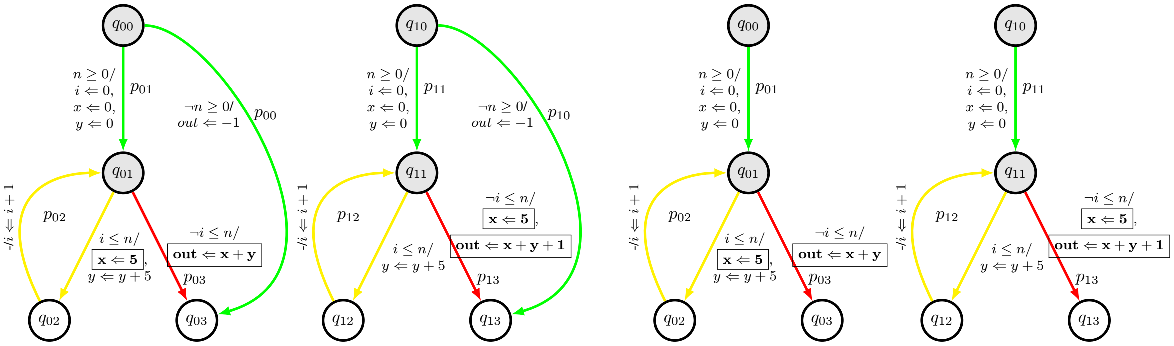
Suppose the source behavior and the transformed behavior are represented as FSMDs M_0 and M_1, respectively. Let us assume that the PBEC fails to find an equivalent for the path α of M_0. We now discuss how to generate a unique computation starting from the reset state that leads to the path α. It may be noted that the EVP method maintains two lists: `EQ_LIST` contains equivalent path pairs explored so far and `C_LIST` contains candidates for conditionally equivalent path pairs. In the EVP method `C_LIST` is obtained in a depth first search (DFS) manner. So, if we traverse backward from the start state of α, we will obtain a sequence of paths from the set `C_LIST`. *This trace would always be a unique trace.* Let the sequence be $\langle p_{0j}, p_{0j+1}, \ldots, p_{0k}, \alpha \rangle$ in FSMD M_0. The segment of the FSMD M_0 from the reset state q_{00} to the start state of p_{0j}, (say p^s_{0j},) is already proved to be equivalent to its corresponding part in FSMD M_1. However, there may be many paths from q_{00} to p^s_{0j}. For our purpose, we can choose one of the paths from this segment. Let us choose the sequence $\langle p_{00}, p_{01}, \ldots, p_{0i} \rangle$ where p_{00} starts from the state q_{00} and the path p_{0i} ends at p^s_{0j}. Therefore, the sequence *cTrace* $= \langle p_{00}, p_{01}, \ldots, p_{0i}, p_{0j}, p_{0j+1}, \ldots, p_{0k}, \alpha \rangle$ is the *cTrace* in the FSMD M_0 that we are interested in. From `EQ_LIST`, we will obtain the paths corresponding to $p_{00}, p_{01}, \ldots, p_{0i}$ in FSMD M_1. Let the corresponding paths be $p_{10}, p_{11}, \ldots, p_{1i}$, respectively. Similarly, the corresponding paths of $p_{0j}, p_{0j+1}, \ldots, p_{0k}$ in the FSMD M_1 can be found using `C_LIST`. Let the corresponding paths be $p_{1j}, p_{1j+1}, \ldots, p_{1k}$, respectively. The *potential* corresponding path of α can also be obtained in the FSMD M_1; let it be β. The EVP method identifies the potential candidate for equivalence, β, in M_1 in most of the cases (see [8] for details). It fails to find β only if there does not exist any path from the corresponding state in M_1 whose condition of execution matches even partially with that of α. In this case, we can take any path from the corresponding state in M_1. Therefore, the corresponding $cTrace$ in FSMD M_1 is $\langle p_{10}, p_{11}, \ldots, p_{1i}, p_{1j}, p_{1j+1}, \ldots, p_{1k}, \beta \rangle$.

Example 1. *Consider the input behavior M_0 and its transformed behavior M_1 shown in Fig. 1. The operation $x \Leftarrow 5$, a loop invariant for input behavior M_0, is placed after the loop body in the transformed behavior M_1. Note that the input behavior M_0 and the transformed behavior M_1, shown in Fig. 1, are not equivalent since there is mismatch in values of the out variable. The EVP method reports that behaviors "May Not be equivalent". The EVP method also reports that the path pairs (p_{00}, p_{10}) and (p_{01}, p_{11}) are U-equivalent, the path pair (p_{02}, p_{12}) is a candidate for C-equivalence and the path pair (p_{03}, p_{13}) is not equivalent. During the course of equivalence checking the EVP method stores these U-equivalent and candidate for C-equivalent path pairs in the* `EQ_LIST` *and* `C_LIST` *list, respectively. As explained in Sec. III, using these lists the generated cTrace of M_0 and M_1 is shown in 1(c) and 1(d), respectively.*

IV. Counter example generation using counter-trace

To obtain the counter-example, i.e., assigning suitable value to the inputs, we rely on CBMC [9] . Specifically, for a given upper bound, CBMC verifies the specified assertions. If any violation of an assertion is detected, a counter-example is generated. Let us consider the *cTraces* as shown in Fig. 1(c) and Fig. 1(d). The input to the CBMC in C for this case is shown in Fig 2.

The variables appearing in the *cTrace* of M_0 (Fig. 1(c)) are suffixed with `_s`,whereas the variables appearing in the *cTrace* of M_1 (Fig. 1(d)) are suffixed with `_t`. Since program equivalence entails identical output(s) generated by the two programs when fed with the same input(s), the input variable n is not suffixed with either `_s` or `_t`. Lines 3 and 4 declare the variables appearing in the *cTrace* of M_0 and the *cTrace* of M_1, respectively, along with their data type which is integer for all the variables. The lines 8–16 and 18–26 capture the data transformations and the conditions of execution of the paths appearing in the *cTrace* of the M_0 and M_1, respectively. We use `__CPROVER_assume` statements to allow only those computation that satisfy a given condition. For example CBMC first picks the value for n non-deterministically from the domain of integers. The


q_{00}
$n \geq 0/$ $i \Leftarrow 0,$ $x \Leftarrow 0,$ $y \Leftarrow 0$
p_{01}
$\neg n \geq 0/$ $out \Leftarrow -1$
p_{00}
q_{01}
$-/i \Leftarrow i+1$
p_{02}
$i \leq n/$ $\boxed{\mathbf{x \Leftarrow 5}},$ $y \Leftarrow y+5$
$\neg i \leq n/$ $\boxed{\mathbf{out \Leftarrow x+y}}$
p_{03}
q_{02}
q_{03}
q_{10}
$n \geq 0/$ $i \Leftarrow 0,$ $x \Leftarrow 0,$ $y \Leftarrow 0$
p_{11}
$\neg n \geq 0/$ $out \Leftarrow -1$
p_{10}
q_{11}
$-/i \Leftarrow i+1$
p_{12}
$i \leq n/$ $y \Leftarrow y+5$
$\neg i \leq n/$ $\boxed{\mathbf{x \Leftarrow 5}},$ $\boxed{\mathbf{out \Leftarrow x+y+1}}$
p_{13}
q_{12}
q_{13}
q_{00}
$n \geq 0/$ $i \Leftarrow 0,$ $x \Leftarrow 0,$ $y \Leftarrow 0$
p_{01}
q_{01}
$-/i \Leftarrow i+1$
p_{02}
$i \leq n/$ $\boxed{\mathbf{x \Leftarrow 5}},$ $y \Leftarrow y+5$
$\neg i \leq n/$ $\boxed{\mathbf{out \Leftarrow x+y}}$
p_{03}
q_{02}
q_{03}
q_{10}
$n \geq 0/$ $i \Leftarrow 0,$ $x \Leftarrow 0,$ $y \Leftarrow 0$
p_{11}
q_{11}
$-/i \Leftarrow i+1$
p_{12}
$i \leq n/$ $y \Leftarrow y+5$
$\neg i \leq n/$ $\boxed{\mathbf{x \Leftarrow 5}},$ $\boxed{\mathbf{out \Leftarrow x+y+1}}$
p_{13}
q_{12}
q_{13}

(a) Source behavior M_0 (b) Transformed behavior M_1 (c) *cTrace* of M_0 (d) *cTrace* of M_1

Fig. 1. Counter-trace Genegartion

```
 1  #include<assert.h>
 2  void main()
 3  {
 4    int i_s,x_s,y_s,n,out_s;
 5    int i_t,x_t,y_t,out_t;
 6    __CPROVER_assume(n>=0);
 7    assert(!(n>=0));
 8    // cTrace for M0
 9    if(n>=0)
10    {
11      i_s=0;x_s=0;y_s=0;
12      __CPROVER_assume(i_s<=n);
13      assert(!(i_s<=n));
14      while(i_s<=n)
15      {
16        x_s=5;
17        y_s=y_s+5;
18        i_s=i_s+1;
19      }
20      out_s=x_s+y_s;
21    }
22    //cTrace for M1
23    if(n>=0)
24    {
25      i_t=0;x_t=0;y_t=0;
26      __CPROVER_assume(i_t<=n);
27      assert(!(i_t<=n));
28      while(i_t<=n)
29      {
30        y_t=y_t+5;
31        i_t=i_t+1;
32      }
33      x_t=5;
34      out_t=x_t+y_t+1;
35    }
36    assert(x_s = x_t);// Live Variable
37    assert(y_s = y_t);// Live Variable
38    assert(out_s = out_t);// Output Variable
39  }
```

Fig. 2. CBMC input for the *cTraces* shown in Fig. 1(c) and Fig. 1(d)

statement `__CPROVER_assume(`$n \geq 0$`)` at line 5 further restricts the range of n for all program computations to be greater than or equal to 0. Note that if there is no computation satisfying the condition, say P, mentioned in `__CPROVER_assume` statement, then all the assertions hold vacuously. We check this by adding `assert(`$!P$`)` statement after each `__CPROVER_assume` statement so that if one of the `assert(`$!P$`)` statement is true then we declare that all the possible computations represented by *cTrace* are false computations [11] i.e., they never execute. Finally, we check the equivalence of the live variables (x_s, y_s, x_t, y_t) and output variables (out_s, out_t) using the `assert` statements (lines `27–29`).

CBMC is able to automatically determine an upper bound on the number of loop iterations in many cases. It may fail if the number of loop iterations is highly data-dependent. Therefore, to verify the assertions with CBMC we use the following command: `cbmc fileName.c -unwind k --no-unwinding-assertions` where *fileName.c* is the name of the target program, k is the bound on the number of iterations of the loop in the program called as Unwinding Loop Bound (ULB) and `--no-unwinding-assertions` disables the unwinding assertion check and changes the unwinding assertion to an unwinding assumption. We use the option `--no-unwinding-assertions` so that a counter-example might be found within the small state space generated with the small ULB. If the target program contains a loop then CBMC unwinds the loop k times and check the properties. Note that if there are multiple loops in the program, the bound k applies to all loops. A violation of the property is reported if it is found within k ULB and CBMC will give a counter-example. Otherwise, we iteratively run CBMC with increasing ULBs for the loops until an assertion violation is found or a given time limit is reached.

V. Incorporation of Results in Equivalence Checking Framework

The PBECs are sound but not complete. Therefore, all the PBECs report that the behaviors "May Not be equivalent" once they fail to prove the equivalence of source and transformed behaviors. Using the output of CBMC, we can actually make the PBEC more powerful. In some scenarios, the PBEC can report that the behaviors are "Not equivalent" (instead of "May Not") along with a counter-example. Also, in some scenarios, the non-equivalence result reported by the PBEC can be proved to be a *false negative* and equivalence checking

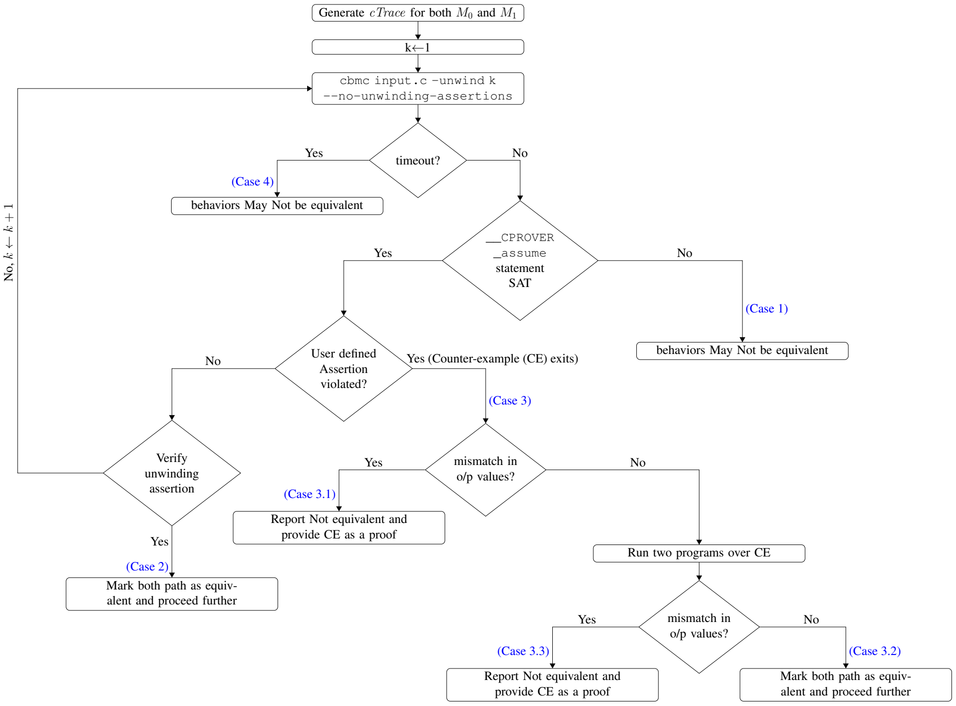

Generate $cTrace$ for both M_0 and M_1
k←1
cbmc input.c -unwind k --no-unwinding-assertions
timeout?
Yes
No
(Case 4)
behaviors May Not be equivalent
__CPROVER _assume statement SAT
Yes
No
(Case 1)
behaviors May Not be equivalent
No, $k \leftarrow k+1$
User defined Assertion violated?
No
Yes (Counter-example (CE) exits)
(Case 3)
Verify unwinding assertion
Yes
(Case 2)
Mark both path as equiv- alent and proceed further
mismatch in o/p values?
Yes
No
(Case 3.1)
Report Not equivalent and provide CE as a proof
Run two programs over CE
mismatch in o/p values?
Yes
No
(Case 3.3)
Report Not equivalent and provide CE as a proof
(Case 3.2)
Mark both path as equiv- alent and proceed further

Fig. 3. Control flow graph of counter-example generation using CBMC and its utilization in a PBEC framework.

will proceed further. In the following, we discuss how we can incorporate the CMBC result to improve the equivalence checking framework.

- **Case 1:** *One of the conditions mentioned in* `__CPROVER_assume` *statement is not satisfiable*: In this case, we report to PBEC that all the possible computations represented by *cTrace* are false computations. Consequently, we need to proceed further in the equivalence checking process.
- **Case 2:** *The unwinding assertions are valid and CBMC does not find any counter-example*: This means the values of all the live variables and output variables are the same for both *cTraces*. So the non-equivalence reported by the PBEC may be a false negative. In this case, we need to proceed further in equivalence checking by declaring the corresponding path pair (α, β) as an equivalent path. This actually helps the PBEC to avoid false negative results during the course of equivalence checking.
- **Case 3:** *CBMC reports counter-example for some variables:* This means the data transformation of some variables is not equivalent in the *cTraces*.
  Case 3.1: *A mismatch is found for an output variable*: This is surely a non-equivalence case. So the equivalence checker correctly found the non-equivalence of the behaviors. In this case, the PBEC reports that the behaviors are "Not equivalent" along with the counter-example.
  If a mismatch is found only for live variables (which are not output variables), then we cannot conclude definitely that the final outputs of both the behaviors will not be the same. There may be some other operations in the subsequent execution of the FSMDs which will make the behaviors equivalent. Therefore, we need to execute the two programs with the counter-example produced by CBMC and check if the outputs of the two programs are the same or not.
  Case 3.2: *The outputs are the same*: This is not a non-equivalent case. Consequently, we need to proceed further in the equivalence checking process.
  Case 3.3: *The outputs of the two programs are not the same*: This is surely a non-equivalence scenario; in this case, the equivalence checker will report the behaviors are "Not equivalent" along with the counter-example.
- **Case 4:** *CBMC hits the time limit:* In this case, CBMC has failed to generate a counter-example because of time out. So no counter-example will be provided to the user. The PBEC reports the behaviors "May Not be equivalent".

VI. Overall Equivalence Checking Framework

The abstract version of our counter-example generation represented by the function `counterExmapleGenerator` is presented in Algorithm 1. The control flow of Algorithm 1 is given in Fig. 3. The function `counterExmapleGenerator` takes as input two FSMDs M_0 and M_1, a path α from the path cover of M_0, a path β from the path cover of M_1, `EQ_LIST` contains equivalent path pairs and `C_LIST` contains candidates for conditionally equivalent path pairs. The function `counterExmapleGenerator` returns $\langle \bar{v}, \mathit{Equiv}, \mathit{falseComp} \rangle$, where $\bar{v} = \langle v_1, v_2, \ldots, v_n \rangle$ is the input variable list such that v_i represents the value of the input variable v_i, *Equiv* is `True` if $\alpha \simeq \beta$ and `False` otherwise and *falseComp* is `True` if all

Algorithm 1: counterExmapleGenerator(M_0, M_1, α, β, EQ_LIST, C_LIST)

```
1  DFS from the start state of $\alpha$ in C_LIST to obtain the sequence
     $\langle p_{0j},\ p_{0j+1},\ \ldots,\ p_{0k},\ \alpha\rangle$.
2  DFS from the start state of $p_{0j}$ in EQ_LIST to obtain the sequence
     $\langle p_{00},\ p_{01},\ \ldots,\ p_{0i}\rangle$.
3  Encode the $cTrace = \langle p_{00}, p_{01}, \ldots, p_{0i}, p_{0j}, p_{0j+1}, \ldots, p_{0k}, \alpha\rangle$ and
     its corresponding cTrace in $M_1$ as C, say "input.c".
4  Initialize the unwinding loop bound (ULB) $k$ to 1.
5  Use cbmc input.c -unwind k --no-unwinding-assertions
     command to invoke CBMC.
6  if The condition mentioned in __CPROVER_assume is not satisfiable then
7      return $\langle$NULL, False, True$\rangle$;                    /* Case 1 */
8  else if All the unwinding assertions along with the user defined assertions are
     valid then
9      return $\langle$NULL, True, False$\rangle$;                    /* Case 2 */
10 else if CBMC produces a counter-example for the assertion belongs to an
     output variable then
11     return $\langle \bar{v}$, False, False$\rangle$;                   /* Case 3.1 */
12 else if CBMC produces a counter-example for the assertion belongs to live
     variable then
13     Execute both $M_0$ and $M_1$ with the values obtained from CBMC as
         inputs.
14     if outputs are the same then
15         return $\langle$NULL, False, False$\rangle$;               /* Case 3.2 */
16     else
17         return $\langle \bar{v}$, False, False$\rangle$;              /* Case 3.3 */
18     end if
19 else if CBMC hits the time limit then
20     return $\langle$NULL, False, False$\rangle$;                   /* Case 4 */
21 else
22     Increase ULB by one (i.e., k=k+1) and go to step 5
23 end if
```

Algorithm 2: correspondenceChecker($M_0, M_1, q_{0i}, q_{1j}, , P_0, P_1, W_{csp}$)

```
1  foreach path $\alpha : (q_{0i} \Rightarrow q_{0m})$ in $P_0$ do
2      if path $\beta : (q_{1j} \Rightarrow q_{1n})$ can be found in $P_1$ such that $\alpha \simeq \beta$ then
3          $W_{csp} = W_{csp} \cup \{(q_{0m}, q_{1n})\}$;
4          Insert $(\alpha, \beta)$ in EQ_LIST.
5      else if path $\beta : (q_{1j} \Rightarrow q_{1n})$ can be found in $P_1$ such that $\alpha \simeq_c \beta$
         then
6          if $q_{0m}$ or $q_{1n}$ is reset state then
7              return failure;
8          else
9              Insert $(\alpha, \beta)$ in C_LIST.
10             correspondenceChecker($M_0, M_1, q_{0m}, q_{1n},$
                 $P_0, P_1, W_{csp}$);
11         end if
12     else
13         $\langle \bar{v}, Equiv, falseComp\rangle \leftarrow$ counterExmapleGenerator($M_0$,
             $M_1$, $\alpha$, $\beta$, EQ_LIST, C_LIST);
14         if $falseComp$ == True then
15             Proceed Further /* Case 1                              */
16         else if $\bar{v} \neq$ NULL then
17             return Not equivalent; /* Case 3.1                      */
18         else if $\bar{v}$ == NULL and $Equiv$ == True then
19             Proceed Further /* Case 2                              */
20         else if $\bar{v}$ == NULL and $Equiv$ == False then
21             Proceed Further /* Case 3.2                            */
22         else
23             return May Not be Equivalent; /* Case 4                  */
24         end if
25     end if
26 end foreach
27 EQ_LIST = EQ_LIST $\cup$ {Last member of C_LIST}
28 C_LIST = C_LIST $\setminus$ {Last member of C_LIST}
29 return success;
```

the computations represented by *cTrace* are false computations and False otherwise. In lines 1–2 of Algorithm 1, a *cTrace* is constructed from the EQ_LIST and C_LIST as discussed in Sec. III. The *cTrace* is encoded as input to CBMC at line 3. The output generated by CBMC may result in various scenarios as discussed in Sec. V. Lines 6–19 of Algorithm 1 handle these cases.

The enhanced version of correspondenceChecker function of the EVP method [8] after incorporating the result of the function counterExmapleGenerator is presented in Algorithm 2. In case of failure, Algorithm 2 invokes the function counterExmapleGenerator (Algorithm 1) at line 13. It may be noted that the EVP method reports failure under this scenario. If counterExmapleGenerator returns a counter-example (i.e., $\bar{v} \neq$ NULL) then the function correspondenceChecker returns "Not equivalent" i.e., the two FSMDs are not equivalent (line 17). If CBMC hits the time limit then we cannot decide whether M_0 is equivalent to M_1. Hence the function correspondenceChecker returns "May Not be Equivalent" (line 23). If CBMC reports that all the possible computations represented by *cTrace* are false computations (i.e., the variable *falseComp* is True) then the function correspondenceChecker needs to be modified to handle this scenario (line 15). If CBMC finds the mismatch in the values of a live variable but outputs of the two programs are the same then we do not report the counter-example (line 21). To handle this case also correspondenceChecker needs to be modified. If CBMC declares that the path pair (α, β) are equivalent (i.e., the variable *Equiv* is True) then it is a false negative result of the correspondenceChecker function (line 19). The correspondenceChecker function must take some decision to avoid the false negative case in the future.

VII. Experimental Result

We have taken the source code of the EVP method and have implemented our counter-example generation procedure on top of it. Once the EVP method fails to prove the equivalence, a *cTrace* is automatically generated using EQ_LIST and C_LIST of the EVP method as discussed in Sec. III. We have then translated the two corresponding *cTraces* as an input to CBMC. For our experiment, we used CBMC version 5.8 [9]. The benchmarks are taken from [5]. The benchmarks are run on a 1.8 GHz Intel i5 processor with 8 GB of RAM with a timeout limit of 60 seconds. The results of our experimentation are tabulated in Table I. We have manually introduced few changes like addition, multiplication or subtraction of a constant to some of the variables in the benchmarks tabulated in rows 3–6 of Table I so that source and transformed behaviors become non-equivalent. For each benchmark, we have reported the number of paths in the path cover, the number of states in the source and transformed behaviors, the equivalence decision taken by the EVP method and our method, the run time in milliseconds (ms) of the EVP method and our method, and the number of lines in the C program given as an input to CBMC.

For the benchmarks DIFFEQ and LRU, both the EVP method and our method report equivalence which is denoted as 'E' in Table I. The objective is to make sure that our implementation does not have any side effect on the existing method.

TABLE I
EXPERIMENTAL RESULTS

Benchmarks	#Path	#State		Decision		Time (ms)		Lines
		M_0	M_1	EVP	Our	EVP	Our	
DIFFEQ	3	15	9	E	E	25	25	-
LRU	39	33	32	E	E	1038	1038	-
DCT	1	8	16	MNE	NE	85	766	185
PERFECT	7	6	4	MNE	NE	56	227	74
MODN	9	8	9	MNE	NE	66	890	137
GCD	11	8	4	MNE	NE	31	100	97
Test Case [12]	6	5	5	MNE	MNE	20	26	32

In the benchmarks reported in rows 3–6, the EVP method fails to prove the equivalence of source and transformed behaviors. It reports that the behaviors "May Not be equivalent". This is reported as 'MNE' in Table I. In these cases, CBMC finds the mismatch in the values of output variable and generates a suitable counter-example with $k = 2$ loop unwindings. Hence, our method concludes that the behaviors are "Not equivalent" which is denoted as 'NE' in Table I. The time required by our method is a little high compared to the EVP method in the case of non-equivalence as we need to run CBMC on the *cTrace*. This experiment shows that with the help of our counter-example generation scheme a PBEC can take strong decisions about the non-equivalence of behaviors. Moreover, the counter-example provided by the PBEC will help the user to debug the root cause of the non-equivalence.

In our second experiment, we try to explore the false negative scenario of the EVP method. For this purpose, we have taken the example given in [12] and the result is tabulated in row 7 of Table I. This test case involves the inverse operation [12]. For this test case, the EVP method reports that the behaviors "May Not be equivalent". However CBMC does not generate any counter-example and case 2 as discussed in Sec VI arises here. CBMC reports that *cTrace* corresponding to these behaviors are equivalent. Our method still reports "May Not be equivalent" since we have not implemented proceed further scenarios in the EVP. This experiment exposes a false negative case of the EVP method. It would be an interesting future work to enhance the EVP to handle the test cases which involves inverse operations.

VIII. RELATED WORK

The basic path-based equivalence checking of the scheduling step of HLS was first proposed in [2]. The PBEC is further enhanced to handle control structure modification and uniform code motions in [7], non-uniform code motion in [6], speculative code motions in [3], code motions across loops in [5], for better correlation of corresponding paths using machine learning in [4], and to handle false computations and code motions involving loops in [8].

In software model checking [13], if the property of a program fails then model checkers generates a counter-example and helps in understanding the root cause of violation of the property. In this work we utilize the counter-example generation procedure in a PBEC framework to improve its performance.

IX. CONCLUSION

In this paper, we have presented a counter-example generation mechanism for the PBEC reported in [8]. A similar counter-example generation mechanism can also be developed for other reported equivalence checking methods as well. The idea is to reuse the equivalence information of a PBEC to generate a counter-trace efficiently and then use CBMC to generate a counter-example. We have also shown that a path-based equivalence checking method can be further strengthened with the counter-example generation mechanism. As shown in the experiments, the EVP method can take stronger equivalence decision with help of counter-examples. Our counter-example generation mechanism identifies a false negative result of the EVP method. In the future, we plan to enhance the EVP method to handle the 'proceed further' (i.e., false negative cases) scenarios identified by our counter-example generation mechanism.

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